Pricing and regulations for a wholesale electricity market

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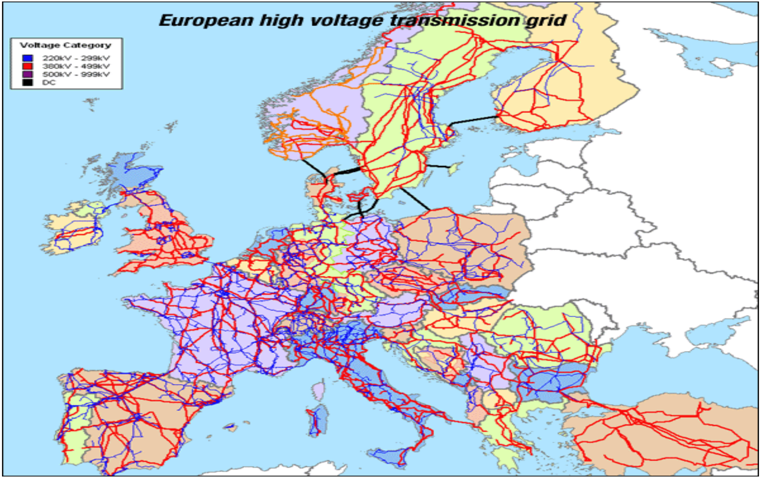
Outline

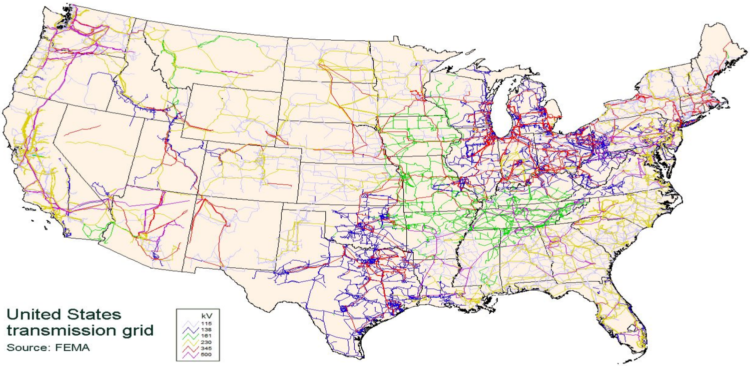
- Introduction and motivation
- Modeling market and Equilibrium.
- Market Power
- Efficient regulations and Extended Mechanism Design
- Conclusions

1. Introduction and motivation
2. Modeling Market
   - Equilibrium: Nash
3. Market Power
4. Efficient regulations and mechanism design
   - The benchmark game
   - Comparing Benchmark with Optimal Mechanism

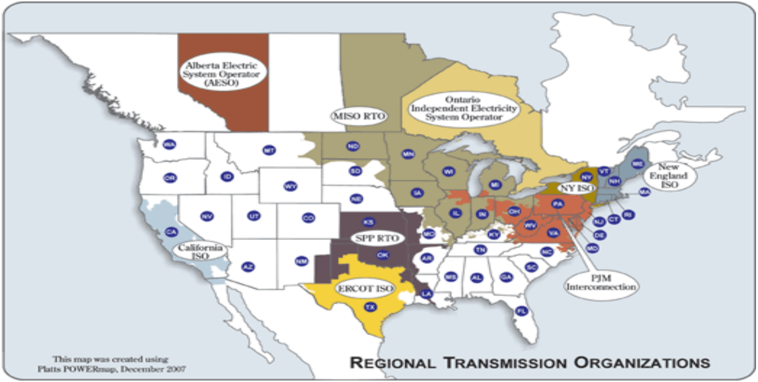
ISO


European high voltage transmission grid
Voltage Category
220kV - 299kV
380kV - 499kV
500kV - 999kV
DC

Transmission US



ISOs USA


Alberta Electric System Operator (AESO)
MISO RTO
Ontario Independent Electricity System Operator
New England ISO
NY ISO
SPP RTO
California ISO
ERCOT ISO
PJM Interconnection
WA
OR
CA
NV
ID
MT
WY
UT
CO
AZ
NM
ND
SD
NE
KS
OK
TX
MN
IA
MO
AR
LA
WI
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TN
MS
AL
GA
FL
SC
NC
VA
WV
PA
NY
VT
NH
ME
MA
CT
RI
NJ
DE
MD
This map was created using Platts POWERmap, December 2007
REGIONAL TRANSMISSION ORGANIZATIONS

A generation short term market: day-ahead mandatory pool

- Today: generators taking into account an estimation of the demand bid *increasing piece-wise linear cost functions or equivalently piece-wise constant "price"*. Even general convex cost functions.
- Tomorrow: the (ISO) using this information and knowing a realization of the demand, minimizes the sum of the costs to satisfy demands at each node considering all the transmission constraints: "dispatch problem".
- Tomorrow: the (ISO) sends back to generators the optimal quantities and "prices" (multipliers associated to supply = demand balance equation at each node)

ISO problem or dispatch $DP(c,d)$

The (ISO) knows a realization of the demand $d \in I\!R^V$, receives the costs functions bid $(c_i)_{i\in G}$ and compute: $(q_i)_{i\in G}$, $(\lambda_i)_{i\in G}$

$$\min_{(h,q)} \quad \sum_{i\in G} c_i(q_i). \tag{1}$$

$$\sum_{e\in K_i} \frac{r_e}{2} h_e^2 + d_i \leq q_i + \sum_{e\in K_i} h_e sgn(e,i), \quad i \in G \tag{2}$$

$$q_i \in [0, \bar{q}_i], \quad i \in G, \tag{3}$$

$$0 \leq h_e \leq \overline{h}_e \tag{4}$$

We denote $Q(c, d) \subset I\!R^G$ the generation component of the optimal solution set associated to each cost vector submitted $c = (c_i)$ and demand d.
We denote $\Lambda(c, d) \subset I\!R^G$ the set of multipliers associated to the supply=demand in the ISO problem.

Modeling Generators

1. At each node $i \in G$ we have a generator with payoff

$$u_i(\lambda, q) = \lambda q - \bar{c}_i(q)$$

   $\bar{c}_i$ is the real cost.

2. The strategic set for each player i denoted S_i:

   $\{c_i : \mathbb{R} \rightarrow \mathbb{R}_+ |$ convex, nondecreasing, bounded subgradients $\}$

   $\partial c_i \subset [0, p^*]$, p^* is a *price cap*.

Equilibrium

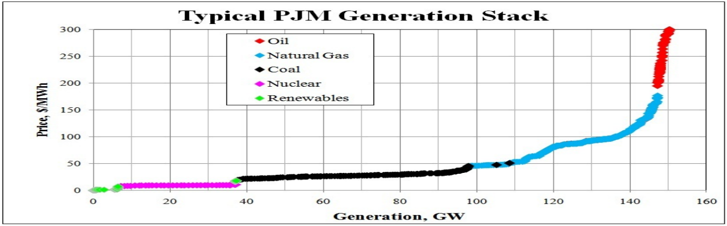
An equilibrium is (q, λ, m) such that q is a selection of $Q(\cdot, \cdot)$ and λ is a selection of $\Lambda(\cdot, \cdot)$ and $m = (m_i)_{i \in G}$ is a mixed-strategy equilibrium of the generator game in which each generator submits costs $c_i \in S_i$ with a payoff

$$\mathbb{E}u_i(\lambda_i(c, \cdot), q_i(c, \cdot)) = \int_D [\lambda_i(c, d) q_i(c, d) - \bar{c}_i(q_i(c, d))] d\mathbb{P}(d),$$

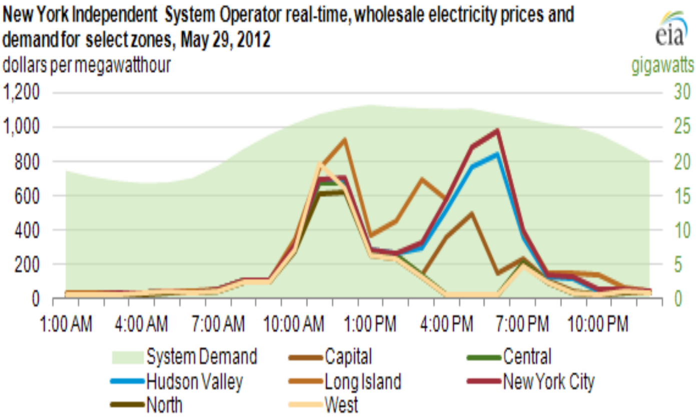
Literature

- In some cases, for example, using a supply function equilibria approach there are previous works by Anderson, Philpott, or using variational inequality approach by Pang, Ralph or also using game theory by Smeers, Wilson, Joskow, Tirole, Oren, Borestein, Bushnel, Wolak...
- Limited network representation or strategic behavior or strategy space.

Generation costs



Prices NY ISO

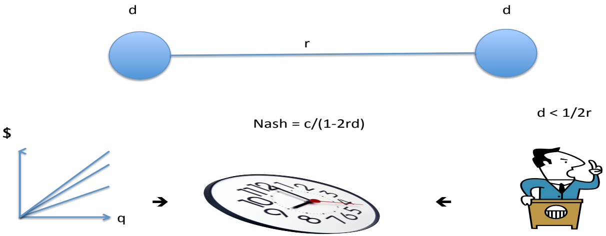


Two-node case

Two nodes case

Symmetric Nash equilibrium

Profit = multiplier × quantity − cost × quantity



the ISO Problem: two-node case

Given that each generator reveals a cost c_i, the (ISO) solves:

$$\begin{array}{ll} \min\limits_{q,h} & \sum\limits_{i=1}^{2} c_i q_i \\ s.t. & q_i - h_i + h_{-i} \geq \frac{r}{2}[h_1^2 + h_2^2] + d \quad \text{for} \quad i = 1, 2 \\ & q_i, h_i \geq 0 \quad \text{for} \quad i = 1, 2 \end{array}$$

Result

- Escobar and J. (ET (2010)) equilibrium exists but producers charge a price above marginal cost:
- $$Nash = \bar{c}/(1 - 2rd)$$

Sensitivity formula

Proposition

Let $c \in \prod_{i \in G} S_i$ *and* $c_i - \hat{c}_i$ *a Lipschitz function with constant* κ*. Then,*

$$|Q_i(c,d) - Q_i(\hat{c}_i, c_{-i}, d)| \leq \kappa\eta,$$

where $\eta = 2\frac{(1+r_i\overline{h}_i)^2}{\min_{i \in G} r_i c_i^+(0)} \in]0, +\infty[$ *and* $c_i^+(0) = \lim_{y \to 0+} \frac{c_i(y) - c_i(0)}{y}$*.*

Why? losses $=>$ the second-order growth

Market Power formula

Dangerous incentive: If the number of generators is small or the topology of the network isolates some demand nodes then the generators will play strategically with the ISO exercising market power.

Market Power formula

Proposition

The equilibrium prices p_i *satisfy*

$$\mathbb{E}|p_i - \gamma| \geq \frac{\mathbb{E}[Q_i(p_i, p_{-i}, d)]}{\bar{\eta}}$$

where $\bar{\eta}_i = 2\frac{|K_i|^2\left(1+\max\{r_e\overline{h}_e \ : \ e\in K_i\}\right)^2}{p_* \min_{e\in K} r_e}$

$\gamma(p_{-i}, d)$ *is a measurable selection of* $\partial\bar{c}_i(Q_i(p_i, p_{-i}, d))$.

Market Power formula

Proposition

Linear case: $\bar{c}_i(q) = \bar{c}_i q$, *then*

$$p_i - \bar{c}_i \geq \frac{\mathbb{E}[Q_i(p_i, p_{-i}, d)]}{\bar{\eta}}.$$

The Questions

In an electric network with **transmission costs** and **private information**:

- Does the usual (price equal Lagrange multiplier) regulation mechanism minimize costs for the society?
- If not, what is the mechanism that achieves this objective?
- How does the performance of both systems compare?

Methodology:

- Bayesian Game Theory
- Mechanism Design

Framework

- Two-node network with demand d at each node.
- One producer at each node, with marginal cost of production $c_i \sim F_i[\underline{c}_i, \overline{c}_i]$.
- Transmission costs rh^2, with h the amount sent from one node to another.

The ISO Problem

Given that each generator reveals a cost c_i, the ISO solves:

$$\begin{aligned} \min_{q,h} \quad & \sum_{i=1}^{2} c_i q_i \\ s.t. \quad & q_i - h_i + h_{-i} \geq \tfrac{r}{2}[h_1^2 + h_2^2] + d \quad \text{for} \quad i = 1, 2 \\ & q_i, h_i \geq 0 \quad \text{for} \quad i = 1, 2 \end{aligned}$$

The Solution for ISO problem

If we define

$$H(x,y) = d + \frac{1}{2r}\left(\frac{x-y}{x+y}\right)^2 - \frac{1}{r}\left(\frac{x-y}{x+y}\right)$$

and

$$\overline{q} = 2\left[\frac{1-\sqrt{1-2dr}}{r}\right]$$

then the solution to this problem can be written as

$$q_i(c_i, c_{-i}) = \begin{cases} H(c_i, c_{-i}) & \text{if } H(c_i, c_{-i}) \geq 0 \text{ and } H(c_{-i}, c_i) \geq 0 \\ \overline{q} & \text{if } H(c_{-i}, c_i) < 0 \\ 0 & \text{if } H(c_i, c_{-i}) < 0 \end{cases}$$

$$\lambda_i(c_i, c_{-i}) \equiv p_i(c_i, c_{-i}) = c_i \quad \text{if } H(c_i, c_{-i}) \geq 0$$

The Bayesian Game

The game:

- 2 players. Strategies $c_i \in C_i = [\underline{c}_i, \overline{c}_i]$, i=1,2.
- Payoff $u_i(c_i, c_{-i}) = (\lambda_i(c_i, c_{-i}) - \mathbf{c}_i) q_i(c_i, c_{-i})$,

where $\mathbf{c}_i$ is the real cost. The Equilibrium:

- A strategy $b_i : [\underline{c}_i, \overline{c}_i] \longrightarrow I\!R^+$ (convex at equilibrium!)
- In a Nash equilibrium

$$\overline{b}(c) \in arg \max_x \int_{C_{-i}} [\lambda_i(x, \overline{b}(c_{-i})) - c] q_i(x, \overline{b}(c_{-i})) f_{-i}(c_{-i}) dc_{-i} \tag{5}$$

Numerical Approximation

- For simplicity $C_i = [1, 2]$.
- Let $k \in \{0, ..., n-1\}$, and $b(c) = b_k$ for $c \in [\frac{k}{n}, \frac{k+1}{n}]$.
- The weight of each interval is given by $w_k = F(\frac{k+1}{n}) - F(\frac{k}{n})$.
- The approximate equilibrium is characterized by:

$$b_k \in arg\max_x \sum_{l=0}^{n-1} [\lambda_i(x, b_l) - r_k] q_i(x, b_l) w_l \quad \text{for all} \quad k \in \{0, ..., n-1\} \tag{6}$$

Optimal Mechanism. Principal Agent Model (Myerson)

- A *direct revelation mechanism* $M = (q, h, x)$ consists of an *assignment rule* $(q_1, q_2, h_1, h_2) : C \longrightarrow R^4$ and a *payment rule* $x : C \longrightarrow R^2$.
- The ex-ante expected profit of a generator of type c_i when participates and declares c_i' is

$$U_i(c_i, c_i'; (q, h, x)) = E_{c_{-i}}[x_i(c_i', c_{-i}) - c_i q_i(c_i', c_{-i})]$$

- A mechanism (q, h, x) is feasible iff:

$$\begin{aligned} U_i(c_i, c_i; (q, h, x)) &\geq U_i(c_i, c_i'; (q, h, x)) \text{ for all } c_i, c_i' \in C_i \\ U_i(c_i, c_i; (q, h, x)) &\geq 0 \text{ for all } c_i \in C_i \\ q_i(c) - h_i(c) + h_{-i}(c) &\geq \frac{r}{2}[h_1^2(c) + h_2^2(c)] + d \text{ for all } c \in C \\ q_i(c), h_i(c) &\geq 0 \text{ for all } c \in C \end{aligned}$$

The Regulator's Problem

Using the revelation principle, the regulator's problem can be written as:

$$\min \int_C \sum_{i=1}^{2} x_i(c) f(c) dc \tag{7}$$

subject to (q, h, x) being "feasible"

The Regulator's Problem (II)

It can be rewritten as

$$\begin{array}{ll} \min & \int_C \sum_{i=1}^{2} q_i(c)[c_i + \frac{F_i(c_i)}{f_i(c_i)}]f(c)dc \\ \text{s.t} & \int_{C_{-i}} q_i(c_i, c_{-i}) f_{-i}(c_{-i}) dc_{-i} \text{ is non-increasing in } c_i \\ & q_i(c) - h_i(c) + h_{-i}(c) \geq \frac{r}{2}[h_1^2(c) + h_2^2(c)] + d \text{ for all } c \in C \\ & q_i(c), h_i(c) \geq 0 \text{ for all } c \in C \end{array}$$

We denote by $J_i(c_i) = c_i + \frac{F_i(c_i)}{f_i(c_i)}$ the virtual cost of agent i. We assume it is increasing (Monotone likelihood ratio property: true for any log concave distribution)

Solution

An optimal mechanism is given by

$$\hat{q}_i(c_i, c_{-i}) = \begin{cases} H(J_i(c_i), J_{-i}(c_{-i})) & \text{if } H(J_i(c_i), J_{-i}(c_{-i})) \geq 0 \text{ ar} \\ \overline{q} & \text{if } H(J_{-i}(c_{-i}), J_i(c_i)) < 0 \\ 0 & \text{if } H(J_i(c_i), J_{-i}(c_{-i})) < 0 \end{cases}$$

$$\hat{x}_i(c_i, c_{-i}) = c_i \hat{q}_i(c_i, c_{-i}) + \int_{c_i}^{\overline{c}_i} \hat{q}_i(s, c_{-i}) ds$$

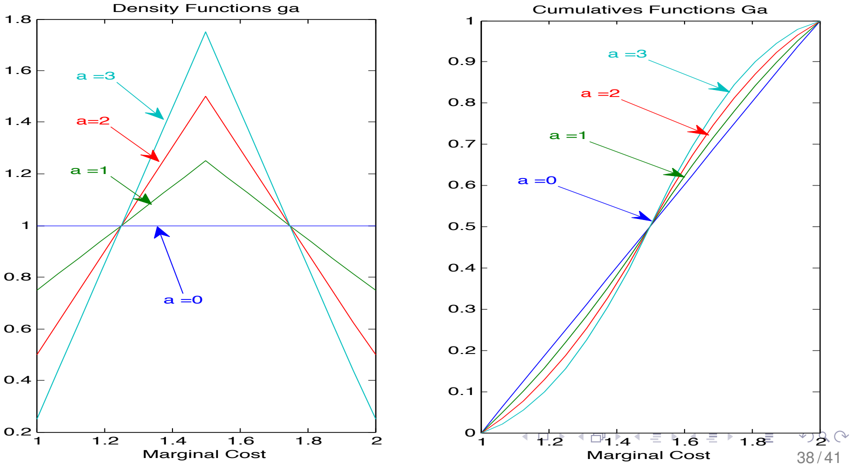
Such a mechanism is dominant strategy incentive compatible.

Comparing Benchmark with Optimal Mechanism

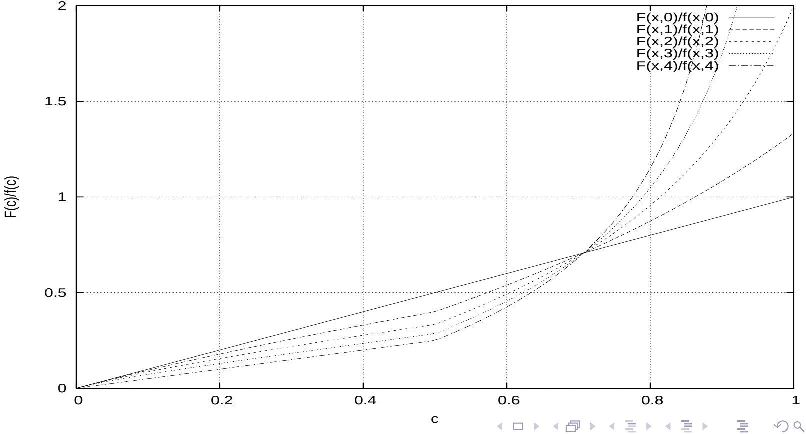
We consider the family of distributions with densities

$$f_a(x) = \begin{cases} a(x-1) + (1 - \frac{a}{4}) & \text{if} \quad x \leq 1.5 \\ -a(x-1) + (1 + \frac{3a}{4}) & \text{if} \quad x \geq 1.5 \end{cases}$$

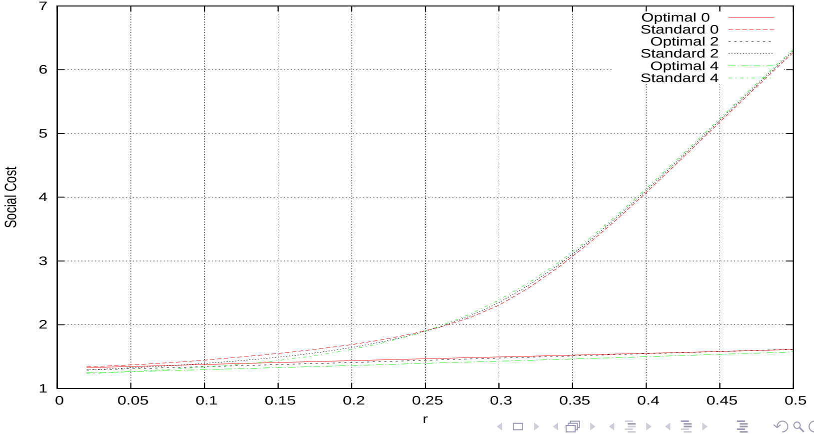
Asymmetric information



Informational rent



Social costs for different mechanisms



Robustness and Practical Implementation

- The optimal mechanism is detail free. If the designer is wrong about common beliefs, then the mechanism is still not bad:

$$||X_f - X_{\tilde{f}}|| \leq ||x||_1 ||f - \tilde{f}||_\infty \leq \bar{c}\bar{q}||f - \tilde{f}||_\infty$$

- The assignment rule is computationally simple to implement. It requires solving **once** the dispatcher problem, with modified costs.
- However, the payments are computationally difficult

$$c_i \hat{q}_i(c_i, c_{-i}) + \int_{c_i}^{\bar{c}_i} \hat{q}_i(s, c_{-i}) ds$$

- Bagh, Adib; Jofre, Alejandro. Weak reciprocally upper-semicontinuity and better reply secure games: A comment. *Econometrica*, Vol. 74 Issue 6 (2006), 1715-1721.
- Bosh, Paul; Jofre, Alejandro; Schultz, Ruediger. Two-Stage Stochastic Programs with Mixed Probabilities. SIAM Optimization (2007) 778-788.
- Escobar, Juan and Jofre, Alejandro. Monopolistic Competition in Electricity Markets. *Economics Theory*, 44, Number 1, 101-121 (2010)
- Escobar, Juan and Jofre, Alejandro. Equilibrium analysis of electricity auctions.
- Escobar, Juan F.; Jofre, Alejandro. Equilibrium analysis for a network market model. Robust optimization-directed

design, 63–72, Nonconvex Optim. Appl., 81, Springer, New York, (2006).