TERRY FEST 2015

May 18-22, Limoges, France

International Conference on Variational Analysis, Optimization and Quantitative Finance

in honor of Terry Rockafellar's 80th birthday

$$\min_{x \in \mathbb{R}^n} f(x) + g(Ax)$$

$$\left(\int_T f(t, \cdot)\, d\mu(t) \right)^* = \int_T f^*(t, \cdot)\, d\mu(t)$$

$$\widehat{\partial}\varphi(\bar{x}) := \left\{ p \in \mathbb{R}^n \ : \ \liminf_{x \to \bar{x}} \frac{\varphi(x) - \varphi(\bar{x}) - \langle p, x - \bar{x} \rangle}{\|x - \bar{x}\|} \geq 0 \right\}$$

$$(Y|X) = \mu_Y + \sigma_Y \sqrt{1 - \rho^2}\, \frac{\varphi(\Phi^{-1}(\alpha))}{1 - \alpha}$$

SCIENTIFIC COMMITTEE:

Samir Adly - University of Limoges, France
Jonathan Borwein - University of Newcastle, Australia
Asen Dontchev - AMS and University of Michigan, USA
Alejandro Jofré - CMM and Universidad de Chile, Santiago, Chile
Roger Wets - University of California, Davis, USA

http://terryfest2015.xlim.fr/

Université de Limoges

institut de recherche

Conception graphique : Linda Jeuffrault CNRS DR8

Dear participants,

On behalf of the University of Limoges and the Laboratory XLIM, I am delighted to welcome all of you to the International Conference on Variational Analysis , Optimization and Quantitative Finance: Terryfest 2015.

The main objective of this conference is to provide a forum for reporting new results and exchanging ideas in the area of variational analysis, optimization and mathematical finance. A wide range of topics will be covered with focus on the following 5 subjects:

1) Variational and Convex Analysis
2) Continuous Optimization
3) Optimal Control Theory
4) Stochastic Programming
5) Risk Management and Quantitative Finance.

This conference is dedicated to Professor R. Tyrrell Rockafellar on the occasion of his 80th birthday, and to honor his exceptional contributions to mathematics and particularly to variational analysis and optimization.

It is a great pleasure and honor for us to organize this conference, and it is our hope that it will be an interesting and learning experience for all of you.

The scientific committee has put together a truly unique programme that addresses the cutting edge in Variational Analysis, Optimization and Quantitative Finance.

Here I would like to express our deepest gratitude to our sponsors and partners for their support.

Many thanks also go to the members of the organizing and scientific committees who made crucial contributions to the success of this conference.

I am confident that you will enjoy your stay in Limoges and that Terryfest 2015 will be an informative and enjoyable event from both the scientific and social points of view. I look forward to spending interesting, exciting and memorable days with you.

My personal respect and thanks go out to all of you.

Samir Adly

Chair of the Organizing Committee

Participants

Confirmed Plenary Speakers

- Martial Agueh, University of Victoria, Canada.
- Hedy Attouch, Université de Montpellier II, France.
- Didier Aussel, Université de Perpignan, France.
- Jean-Bernard Baillon, Université Paris 1, France.
- Gerald Beer, California State University Los Angelos, USA.
- Joseph Frédéric Bonnans, INRIA Saclay Ile-de-France.
- Jean-Marc Bonnisseau, Université de Paris 1, France.
- James Burke, University of Washington, Seattle, USA.
- Patrick Louis Combettes, Université Pierre et Marie Curie, Paris 6, France.
- Roberto Cominetti, Universidad de Chile, Chile.
- Bernard Cornet, Université Paris 1, France.
- Rafael Correa, CMM and Universidad de Chile, Chile.
- Aris Daniilidis, CMM and Universidad de Chile, Chile.
- Asen Dontchev, Mathematical Reviews and University of Michigan, Ann Arbor, USA.
- Dmitriy Drusvyatskiy, University of Washington Seattle, USA.
- Ivar Ekeland, Université Paris-Dauphine, France.
- Jalal Fadili, ENSICAEN, France.
- Fabián Flores-Bazán, University of Concepcion, Chile.
- Hélène Frankowska, Université Pierre et Marie Curie, Paris 6, France.
- Masao Fukushima, Nanzan University, 日本.
- Rafael Goebel, Loyola University Chicago, USA.
- Abderrahim Hantoute, CMM and Universidad de Chile, Chile.
- René Henrion, Weierstrass Institute of Berlin, Deutschland.
- Alexander Ioffe, The Technion, Israel.
- Milen Ivanov, University of Sofia, Bulgaria.
- Alejandro Jofré, CMM and Universidad de Chile, Chile.
- Abderrahim Jourani, Université de Dijon, France.
- Diethard Klatte, University of Zurich, Switzerland.
- Jean Bernard Lasserre, LAAS and IMT Toulouse, France.
- Adrian Lewis, Cornell University, USA.
- Marco Antonio López Cerdá, Universidad de Alicante, España.
- Yves Lucet, University of British Columbia, Canada.
- Russell Luke, University of Göttingen, Deutschland.
- Boris Mordukhovich, Wayne State University, USA.
- Dominikus Noll, University of Toulouse, France.
- Jong-Shi Pang, University of Southern California, USA.
- Jean-Paul, Penot, Université Pierre et Marie Curie, France.
- Teemu Pennanen, King's college London.
- Michael L. Overton, Courant Institute of Mathematical Sciences,
- New York University, USA.
- Stephen M. Robinson, University of Wisconsin-Madison, USA.
- Johannes O. Royset, Naval Postgraduate School, Monterey, California, USA.
- Claudia A. Sagastizabal, Instituto de Matematica Pura e Aplicada, Brazil.
- Mikhail V. Solodov, Instituto de Matematica Pura e Aplicada, Brazil.
- Sylvain Sorin, Université Pierre et Marie Curie, Paris 6, France.
- Jie Sun, Curtin University, Australia.
- Lionel Thibault, Université de Montpellier II, France.
- Nizar Touzi, Ecole polytechnique, France.
- Stan Uryasev, University of Florida, USA.
- Richard Vinter, Imperial College, London, UK.
- Shawn Wang, University of British Columbia, Canada.
- Roger Wets, University of California, Davis, USA.
- Peter Wolenski, Louisiana State University, USA.
- Jane Ye, University of Victoria, Canada.

Confirmed Attendees

- Mario Bravo, Universidad de Chile, Chile.
- Sandrine Charousset, EDF Clamart, Paris, France.
- Emilio Vilches, Institut de Mathématiques de Bourgogne, France.
- Rita Pini, Università degli Studi Milano Bicocca, Italy.
- Monica Bianchi, Università Cattolica del Sacro Cuore, Milano, Italy.
- Noureddine Lehdili, Research and Development NATIXIS.
- Andrea Giovanni Calogero, Università degli Studi Milano Bicocca, Italy.
- David Salas, Université Montpellier 2, France.
- Benjamin Heymann, INRIA Saclay, France.
- Mihail Hamamdzhiev, University of Sofia, Bulgaria.
- David Salas, University of Montpellier 2, France.
- Nadia Zlateva, University of Sofia, Bulgaria.
- Ari-Pekka Perkkiö, Technische Universtität Berlin, Germany.
- Luis Briceno, Universidad Técnica Federico Santa María, Chile.
- Guillaume Garrigos, University of Montpellier 2, France.
- Joon Kwon, Université Pierre-et-Marie-Curie, Paris 6, France.
- Tangi Migot, INSA Rennes, France.
- Iman Mehrabi Nezhad, University of Milano-Bicocca, Italy.
- Sorin-Mihai Grad, Chemnitz University of Technology, Germany.
- Piernicola Bettiol, University of Brest, France.
- Pierre Carpentier, ENSTA-Paristech, France.
- Chirifi Ousri, ENSTA-Paristech, France.
- Vincent Leclere, ENPC-CERMICS, France.
- Aleksandr Aravkin, University of Washington, USA.
- Miroslav Pistek, CNRS-PRMES, France.
- Cesare Molinari, Universidad Técnica Federico Santa Maria and BCAM, Chile.
- Taron Zakaryan, University of Chile.
- Juan Peypouquet, Universidad Técnica Federico Santa Maria, Chile.
- Alberto Zaffaroni, Università di Modena e Reggio Emilia, Italy.
- Victor Riquelme, INRIA, France.
- Saverio Salzo, Università degli Studi di Genova, Italy.
- Samir Adly, University of Limoges, France.
- Paul Armand, University of Limoges, France.
- Moulay Barkatou, University of Limoges, France.
- Joël Benoist, University of Limoges, France.
- Poala Boito, University of Limoges, France.
- Loïc Bourdin, University of Limoges, France.
- Francisco Silva, University of Limoges, France.
- Noureddine Igbida, University of Limoges, France.
- Vincent Jalby, University of Limoges, France.
- Henri Massias, University of Limoges, France.
- Michel Théra, University of Limoges, France.
- Olivier Prot, University of Limoges, France.
- Olivier Ruatta, University of Limoges, France.
- Florent Nacry, University of Limoges, France.
- Van Vu Nguyen, University of Limoges, France.
- Isaï Lankoandé, University of Limoges, France.
- Van Thanh Nguyen, University of Limoges, France.

Schedule Overview

	Monday	Tuesday	Wednesday	Thursday	Friday
8:00 - 8:30	Registration				
	Chairman: S. Adly	*Chairman:* M. Thera	*Chairman:* J.P. Penot	*Chairman:* S. Robinson	*Chairman:* H. Frankowska
8:30 - 9:00	Welcome Ceremomy	B. Mordhukovich	A. Ioffe	J. Burke	
9:00 - 9:30	R. Wets	L. Thibault	D. Klatte	J.B. Baillon	S. Uryasev
9:30 - 10:00	S. Robinson	M. Lopez	G. Beer	P.L. Combettes	B. Cornet
10:00 - 10:30	H. Attouch	J. Ye	S. Sorin	J.S. Pang	J.M. Bonnisseau
10:30 - 11:00	Coffee break	Coffee break	Coffee break	Coffee break	Coffee break
	Chairman: A. Jofré	*Chairman:* R. Vinter	*Chairman:* J. S. Pang	*Chairman:* L. Thibault	*Chairman:* J.M. Bonnisseau
11:00 - 11:30	J.P. Penot	P. Wolenski	H. Frankowska	D. Noll	N. Touzi
11:30 - 12:00	R. Cominetti	A. Lewis	R. Vinter	D. Aussel	A. Jofré
12:00 - 12:30	M. Overton	M. Fukushima	A. Dontchev	R. Henrion	I. Ekeland
	Lunch	**Lunch**	**Lunch**	**Lunch**	**Lunch**
	Chairman: H. Attouch	*Chairman:* J. Ye		*Chairman:* P.L. Combettes	
14:00 - 14:30	A. Daniilidis	T. Pennanen		R. Goebel	
14:30 - 15:00	M. Agueh	J. Royset		M. Ivanov	
15:00 - 15:30	A. Jourani	R. Luke		Y. Lucet	
15:30 - 16:00	F. Flores-Bazan	D. Drusvyatskiy		A. Hantoute	
16:00 - 16:30	Coffee break	Coffee break	**Excursion**	Coffee break	
	Chairman: B. Mordukhovich	*Chairman:* R. Correa		*Chairman:* C. Sagastizabal	
16:30 - 17:00	F.J. Bonnans	M. Solodov		J.B. Lasserre	
17:00 - 17:30	C. Sagastizabal	J. Sun		X. Wang	
17:30 - 18:00	R. Correa	J. Fadili			

Tuesday 19:00 – 20:00: Reception Town Hall

Thursday 19:00 – 00:00: Banquet

Program

Monday, May 18th 2015	*Amphithéâtre Lombois*
8:00	*Registration*
8:30	*Welcome ceremony*
	Chairman: S. ADLY
9:00	**Roger WETS** (University of California, Davis) "A tale about functions of two variables"
9:30	**Stephen ROBINSON** (University of Wisconsin-Madison) "Projections and the Reduction Lemma"
10:00	**Hedy ATTOUCH** (Université Montpellier 2) "Fast convergence of an inertial dynamics with vanishing viscosity in convex optimization"
10:30	*Coffee break*
	Chairman: A. JOFRE
11:00	**Jean-Paul PENOT** (Laboratoire Jacques-Louis Lions, UPMC Paris 6) "Regularity and regularization in variational analysis"
11:30	**Roberto COMINETTI** (Universidad de Chile) "Recursive Optimal Transport and Fixed-Point Iterations for Nonexpansive Maps"
12:00	**Michael OVERTON** (New York University) "Investigation of Crouzeix's Conjecture via Nonsmooth Optimization"
12:30	*Lunch*
	Chairman: H. ATTOUCH
14:00	**Aris DANIILIDIS** (DIM-CMM, Universidad de Chile) "Nonsmooth critical values and Sard type results"
14:30	**Martial AGUEH** (University of Victoria) "A class of total variation minimization problems"
15:00	**Abderrahim JOURANI** (Université de Bourgogne) "Positively α-far sets and existence results for generalized perturbed sweeping processes"
15:30	**Fabian FLORES-BAZAN** (Universidad de Concepción) "Geometric and topological characetrizations of strong duality in nonconvex optimization"
16:00	*Coffee break*
	Chairman: B. MORDUKHOVICH
16:30	**J Frédéric BONNANS** (Inria-Saclay and CMAP, Ecole Polytechnique) "Second Order Analysisfor Optimal Control Problems with Singular Arcs"
17:00	**Claudia SAGASTIZABAL** (IMPA [visiting researcher], Rio de Janeiro, Brazil) "An approximation scheme for a class of risk-averse stochastic equilibrium problems"
17:30	**Rafael CORREA** (CMM - Universidad de Chile) "Integration and approximate subdifferentials calculus for nonconvex functions"

Tuesday, May 19th 2015	*Amphithéâtre Lombois*
	Chairman: M. THERA
8:30	**Boris MORDHUKOVICH** (Wayne State University) "Full Stability of Parametric Variational Systems"
9:00	**Lionel THIBAULT** (Université Montpellier 2) "Three Rockafellar Theorems"
9:30	**Marco LOPEZ** (Alicante University, Alicante, Spain) "Outer limit of subdifferentials and calmness moduli in linear and nonlinear programming"
10:00	**Jane YE** (University of Victoria) "Necessary optimality conditions for optimal control problems with equilibrium constraints"
10:30	*Coffee break*
	Chairman: R. VINTER
11:00	**Peter WOLENSKI** (Louisiana State University) "Essential velocities in stratified control systems"
11:30	**Adrian LEWIS** (Cornell University) "Generic sensitivity analysis for semi-algebraic optimization"
12:00	**Masao FUKUSHIMA** (Nanzan University) "Squared Slack Variables in Nonlinear Second-Order Cone Programming"
12:30	*Lunch*
	Chairman: J. YE
14:00	**Teemu PENNANEN** (King's College London) "Duality and optimality in stochastic optimization and mathematical finance"
14:30	**Johannes ROYSET** (Naval Postgraduate School, Monterey, California) "Measures of Residual Risk with Connections to Regression, Risk Tracking, Surrogate Models, and Ambiguity"
15:00	**Russell LUKE** (Georg-August-Universität Göttingen) "A Survey of Results on Linear Convergence for Iterative Proximal Algorithms in Nonconvex Settings"
15:30	**Dmitriy DRUSVYATSKIY** (University of Washington) "Tame Variational Analysis"
16:00	*Coffee break*
	Chairman: R. CORREA
16:30	**Mikhail SOLODOV** (IMPA, Rio) "Newton-Type Methods: A Broader View"
17:00	**Jie SUN** (Curtin University, Australia) "A distribution-ambiguous scheme for multi-stage stochastic optimization"
17:30	**Jalal FADILI** (CNRS-ENSICAEN-Université de Caen, France) "Finite identification and local linear convergence of proximal splitting algorithms"

Wednesday, May 20th 2015	***Amphithéâtre 250***
	Chairman: J.-P. PENOT
8:30	**Alexander IOFFE** (Technion) "Quadratic growth and subregularity of subdifferentials"
9:00	**Diethard KLATTE** (University of Zurich) "Calm and Locally Upper Lipschitz Multifunctions: Intersection Mappings and Applications in Optimization"
9:30	**Gerald BEER** (California State University, Los Angeles) "On Locally Lipschitz Functions"
10:00	**Sylvain SORIN** (UPMC-Paris 6) "Finite composite games: equilibria and dynamics"
10:30	*Coffee break*
	Chairman: J.S. PANG
11:00	**Hélène FRANKOWSKA** (CNRS and Institut de Mathématiques de Jussieu - Paris Rive Gauche, UPMC, Paris) "Pointwise Second Order Optimality Conditions in Optimal Control"
11:30	**Richard VINTER** (Imperial College London) "Regularity Properties of the Hamiltonian in Optimal Control"
12:00	**Asen DONTCHEV** (AMS and Univ. of Michigan) "ω-limit sets for differential inclusions"
12:30	*Lunch*

From 14:00 to 16:30, visit Limoges, town of ceramic, art and history

Departure: in front of the Faculty of Law and Economics

DISCOVER LIMOGES, AND ITS HISTORICAL QUARTERS ON BOARD THE LITTLE TRAIN !
During one hour, the little train will lead you through streets and lanes to discover the main sites and monuments of the city. Available languages : French, English, Dutch, Italian, Spanish, German, Japanese.

VISIT THE CASSEAUX FORMER PORCELAIN KILN
Less than 500 metres upstream from Saint-Etienne bridge, the Casseaux Former Porcelain Kiln is one of the last witnesses of the "red" city's 130 kilns and the only one available for visits. Dating from the 19th century and listed Historic Monument, it remains the symbol of the uncertain mastery of fire in the art of porcelain. Guided tour in English language

Thursday, May 21th 2015	*Amphithéâtre 250*
	Chairman: S. ROBINSON
8:30	**Jim BURKE** (University of Washington) "Level Set Methods in Convex Optimization"
9:00	**Jean-Bernard BAILLON** (SAMM Université Paris 1 Panthéon-Sorbonne) "My first meeting with the mathematical work of Terry"
9:30	**Patrick COMBETTES** (Université Pierre et Marie Curie) "Horizontal and Vertical Block Decomposition of Monotone Operator Splitting Algorithms"
10:00	**Jong-Shi PANG** (University of Southern California) "Stochastic Non-Cooperative Games"
10:30	*Coffee break*
	Chairman: L. THILBAULT
11:00	**Dominikus NOLL** (University of Toulouse, France) " Non-smooth bundle trust-region method"
11:30	**Didier AUSSEL** (University of Perpignan, France) "Limiting normal approach in quasiconvex analysis"
12:00	**René HENRION** (Weierstrass Institute, Berlin, Germany) "Calmness as a constraint qualification for MPECs"
12:30	*Lunch*
	Chairman: P.L. COMBETTES
14:00	**Rafal GOEBEL** (Loyola University Chicago) "Conjugate Duality and Linear Dynamics with Constraints or Uncertainty"
14:30	**Milen IVANOV** (Sofia University) "New Proofs of Maximal Monotonicity and Integrability of the Subdifferential of Convex Function"
15:00	**Yves LUCET** (University of British Columbia) "On the convexity of piecewise-defined functions"
15:30	**Abderrahim HANTOUTE** (CMM - Universidad de Chile) "On the subdifferential mapping of convex integral functions"
16:00	*Coffee break*
	Chairman: C. SAGASTIZABAL
16:30	**Jean LASSERRE** (LAAS-CNRS and Institute of Mathematics, University of Toulouse, France) "Reconstruction of algebraic-exponential data from moments"
17:00	**Xianfu WANG** (Department of Mathematics, University of British Columbia, Kelowna, Canada) "Resolvent Averages of Monotone Operators in Hilbert Spaces"

Friday, May 22th 2015	*Amphithéâtre 250*
	Chairman: H. FRANKOWSKA
9:00	**Stan URYASEV** (University of Florida) "Buffered Probability of Exceedance: Mathematical Properties and Applications"
9:30	**Bernard CORNET** (Paris School of Economics, Université Paris 1 and University of Kansas, Lawrence) "Submodular financial markets with frictions"
10 :00	**Jean-Marc BONNISSEAU** (Paris School of Economics, Université Paris 1 Panthéon-Sorbonne) "A note on the characterization of optimal allocations in OLG economies with multiple goods"
10:30	*Coffee break*
	Chairman: J.M. BONNISSEAU
11:00	**Nizar TOUZI** (Ecole Polytechnique, Paris) "Martingale Optimal Transport and Robust Hedging"
11:30	**Alejandro JOFRE** (CMM & Universidad de Chile) "Cost-minimizing regulations for an electricity market"
12:00	**Ivar EKELAND** (University Paris Dauphine) " Convex analysis in action: demand functions"
12:30	*Lunch*

Menus

Monday 18

- Joues de porc confites en aspic
- Sauté de veau aux champignons
- Entremet aux fraises
- Vins blancs et rouges
- Café

Tuesday 19

- Mousse de thon et betterave avec son mesclun
- Chartreuse de porc à la moutarde à l'ancienne
- Gâteau aux 3 chocolats
- Vins blancs et rouges
- Café

Wednesday 20

- Saumon mariné aux herbes cuit au sel de Guérande
- Suprême de poulet à l'estragon
- Tarte citron
- Vins blancs et rouges
- Café

Thursday 21

- Lunch

- Terrine de courgettes
- Pavé de bœuf sauce vin rouge
- Baba aux fruits
- Vins blancs et rouges
- Café

- Banquet

- Apéritif : Salé froid – Kir Limousin
- Tatin de saint-jacques aux légumes croquants
- Noix de veau braisée crème de morilles Ou pavé de sandre à l'huile de truffes
- Fromages 3 variétés (en plateau)
- Gâteau aux trois chocolats et framboisier.
- Café
- Vins
    - Brumont Blanc sec, IGP Côtes de Gascogne
    - Château Lamothe Cissac 2008 : Haut-médoc Cru Bourgeois
    - Champagne V. Renard de Beaumont Brut

Friday 22

- Melon et jambon cru
- Dos de saumon et gâteau de pommes de terre
- Duo abricot-vanille
- Café
- Vins blancs et rouges

List of Speakers

A class of total variation minimization problems

Martial Agueh
University of Victoria

Guillaume Carlier
CEREMADE, Université Paris Dauphine

Keywords: Total variation, Rearrangement, Sharp inequalities, 1-Laplacian PDEs.

Motivated by the sharp L^1 Gagliardo-Nirenberg inequality, we prove by elementary arguments that given two increasing functions F and G, solving the total variation minimization problem

$$\inf \left\{ E_{\pm}(u) = \int_{\mathbb{R}^n} \mathrm{d}|\nabla u| \pm \int_{\mathbb{R}^n} F(|u|) \ : \ \int_{\mathbb{R}^n} G(|u|) = 1 \right\}$$

amounts to solve a one-dimensional optimization problem. Under appropriate conditions on the nonlinearities F and G, we showed that the infimum is attained and the minimizers are multiple of characteristic functions of balls. Several variants and applications are discussed, among which some sharp inequalities and nonexistence and existence results to some partial differential equations involving the 1-Laplacian.

Fast convergence of an inertial dynamics with vanishing viscosity in convex optimization. Link with Nesterov algorithm.

Hedy Attouch
Université Montpellier 2, France

Juan Peypouquet
Univesidad Técnica Federico Santa Maria, Valparaiso, Chile

Patrick Redont
Université Montpellier 2, France

Keywords: Convex optimization, fast convergent methods, dynamical systems, gradient flows, inertial dynamics, vanishing viscosity, Nesterov method.

In a Hilbert space setting $\mathcal{H}$, we study the fast convergence properties as $t \to +\infty$ of the trajectories of the second-order evolution equation

$$\ddot{x}(t) + \frac{\alpha}{t}\dot{x}(t) + \nabla\Phi(x(t)) = 0,$$

where $\nabla\Phi$ is the gradient of a convex continuously differentiable function $\Phi : \mathcal{H} \to \mathbb{R}$, and α is a positive parameter. In this damped inertial system, the viscous damping coefficient $\frac{\alpha}{t}$ vanishes asymptotically, but not too fast, see [1]. For $\alpha > 3$, just assuming that $\operatorname{argmin}\Phi \neq \emptyset$, we show that any trajectory converges weakly to a minimizer of Φ. The strong convergence is established in various practical situations. These results complete the fast convergence of the values

$$\Phi(x(t)) - \min\Phi \leq \frac{C}{t^2}$$

obtained by Su, Boyd and Candès [3], and give a continuous version of the convergence results of Chambolle-Dossal [2]. When the solution set is empty, we show that the minimizing property still stands for $\alpha > 1$, but the rapid convergence of values may not be satisfied. Time discretization of this system provides new fast converging algorithms, expanding the field of rapid methods for structured convex minimization initiated by Nesterov, and Beck-Teboulle (FISTA).

References

[1] A. CABOT, H. ENGLER, S. GADAT, *On the long time behavior of second-order differential equations with asymptotically small dissipation* Trans. AMS, **361** (2009), pp. 5983–6017.

[2] A. CHAMBOLLE, CH. DOSSAL, *On the convergence of the iterates of Fista*, HAL Id: hal-01060130 https://hal.inria.fr/hal-01060130v3 (2014).

[3] W. SU, S. BOYD, E. J. CANDÈS, *A differential equation for modeling Nesterov's accelerated gradient method: theory and insights*. Advances in Neural Information Processing Systems **27** (NIPS 2014).

Limiting normal approach in quasiconvex analysis

Didier Aussel
University of Perpignan, France

M. Pištěk
The Institute of Information Theory and Automation of the CAS,
Prague, Czech Republic

Keywords: Quasiconvex optimization, Normal operator, Limiting sublevel set.

A new approach to quasiconvex analysis is proposed here with a clear relation to modern nonsmooth variational analysis.
Inspired by similar definition in subdifferential theory, we define limiting sublevel set and limiting normal operator maps for quasiconvex functions. These maps satisfy interesting properties as semicontinuity and quasimonotonicity. Moreover, calculus rules together with necessary and sufficient optimality conditions for constrained optimization are established.

References

[1] D. AUSSEL & M. PISTEK, *Limiting Normal Operator in Quasiconvex Analysis*, submitted, 2014, 18 pp.

[2] B.S. MORDUKHOVICH, *Variational analysis and generalized differentiation. I: Basic theory. II: Applications.*, Grundlehren der Mathematischen Wissenschaften 330/331. Berlin: Springer. xxii, 579 p., xxii, 610 p., 2006.

My first meeting with the mathematical work of Terry

Jean-Bernard Baillon
SAMM Université Paris 1 Panthéon-Sorbonne

Keywords: convex function, nonlinear maximal monotone operator, suddifferential.

When I was a PhD student, I made research (with G. Haddad) about a question of H. Brézis. In this occasion, I discover the work of Terry (Characterization of subdifferentials of convex functions). We made an article about n-cyclically monotone operators. I present the actual research of this topic.

References

[1] R. T. ROCKAFELLAR, *Characterization of the subdifferentials of convex functions*, Pacific J. Math. 17, 497–510, 1966.

[2] J.-B. BAILLON, G. HADDAD, *Quelques propriétés des opérateurs angle-bornés et n-cycliquement monotones*, Israel J. Math. 26, 137–150 , 1977.

[3] H. H. BAUSCHKE, P. L. COMBETTES, *The Baillon-Haddad theorem revisited*, Journal of Convex Analysis, vol. 17, no. 4,781-787, 2010.

[4] C. L. BYRNE, *On A Generalized Baillon-Haddad Theorem for Convex Functions on Hilbert Space*, to appear in Journal of Convex Analysis.

On Locally Lipschitz Functions

Gerald Beer
California State University, Los Angeles

M. Isabel Garrido
Complutense, Madrid

Keywords: locally Lipschitz function, uniformly locally Lipschitz function, Lipschitz in the small function, uniform approximation, UC-space, cofinally complete space.

The class of locally Lipschitz functions defined on an arbitrary metric space is uniformly dense in the real-valued continuous functions defined on the space. A properly smaller class of locally Lipschitz functions, the class of Lipschitz in the small functions introduced by Luukkainen, is uniformly dense in the uniformly continuous real-valued functions. An optimal proof of the first is based on Lipschitz partitions of unity as identified by Frolik, and for the second an optimal proof is based on a regularization of the initial uniformly continuous function by appropriate smoothing kernels. Between the continuous functions and the uniformly continuous functions sits the class of functions mapping Cauchy sequences to Cauchy sequences, containing within them those locally Lipschitz functions where balls about each point of a common radius exist on which the function is Lipschitz. While a parallel uniform density result is not valid, we give necessary and sufficient conditions on the domain space for uniform density. We also present necessary and sufficient conditions for pairwise coincide of our three classes of locally Lipschitz functions, and identify the common sets of boundedness for each function class.

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Second Order Analysis for Optimal Control Problems with Singular Arcs

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Keywords: Optimal control, partial differential equations, singular arc, second order optimality conditions, Goh transform.

We consider optimal control problems for which the control enters linearly in the cost function and dynamics. They may have singular arcs (over which the constraints are non active). Then the Goh transform allows to obtain specific second order (necessary or sufficient) optimality conditions. In this talk we will review some recent progress of the theory.

1. Presence of state constraints [2]. Assuming the control and state constraint to be scalar, the state constraint being of order one, we show how to obtain second order necessary conditions by adapting the Goh transform technique to this case. We need to assume the discontinuity of the control at junction points. Applications include the Goddard ascent problem.

2. Extension to the optimal control of PDEs setting. The case of a semilinear heat equation is analyzed in [3]. We apply the Goh transform technique and obtain an extension of the Goh-Legendre condition. A numerical study supports the claim for the existence of a singular arc. Preliminary results will be presented in the case of the Schrödinger equation [3], where the control is through a magnetic field and therefore enters naturally in a linear way.

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A note on the characterization of optimal allocations in OLG economies with multiple goods

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Keywords: Overlapping generations model, preference set, normal cone, equilibrium, Pareto optimality.

We consider a pure exchange overlapping generations economy with finitely many commodities and consumers per period having possibly non-complete non transitive preferences. We provide a geometric and direct proof of the Balasko-Shell characterization of Pareto optimal allocation (See, [1]).

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Level Set Methods in Convex Optimization

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Keywords: convex duality, the perspective mapping, extended linear-quadratic programming, Newton's method, complexity.

We present a framework for solving convex optimization problems by successively optimizing over the level sets of the objective. This is done by exchanging the objective with one of the constraint functions and then studying properties of the resulting optimal value function. The approach has classical origins and is the basis for the SPGL1 algorithm of Van den Berg and Friedlander. Several numerical illustrations are presented. In addition, we present new results on the complexity of the method for a range of applications.

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Horizontal and Vertical Block Decomposition of Monotone Operator Splitting Algorithms

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Keywords: Block-coordinate algorithm, block-iterative algorithm, fixed-point algorithm, monotone operator splitting, stochastic quasi-Fejér sequence, stochastic algorithm, structured convex minimization problem

Monotone operator splitting technology constitutes the theoretical and algorithmic foundation of a wide array of numerical methods in data-driven problems. Fueled by new developments in abstract duality for monotone inclusions and product space techniques, significant advances have been made in splitting methods in recent years. In particular, it is now possible to solve highly structured monotone inclusions with algorithms which guarantee the convergence of the iterates. In problems of huge sizes, the implementation of these algorithms faces significant challenges which often render them inapplicable. We present two approaches to face this issue, which both preserve the splitting and convergence properties of the algorithms. First (joint work with J.-C. Pesquet), we propose a general stochastic block-coordinate fixed point framework with arbitrary sampling of the indices of the blocks, from which we derive flexible block-coordinate versions of common splitting algorithms. Second, (joint work with J. Eckstein), we propose a block-coordinate primal-dual splitting framework for composite monotone inclusions in which only subgroups of operators need to be activated at each iteration.

Recursive Optimal Transport and Fixed-Point Iterations for Nonexpansive Maps

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Keywords: Nonexpansive maps, fixed points, Krasnoselskii-Mann iteration, asymptotic regularity

A popular method to compute a fixed point for a non-expansive map $T : C \to C$ is the following successive average iteration originally proposed by Krasnoselskii and Mann

$$(KM) \qquad x_{n+1} = (1-\alpha_{n+1})x_n + \alpha_{n+1}Tx_n.$$

We show how optimal transport can be used to establish a recursive formula to estimate the distance between iterates $\|x_m - x_n\| \leq d_{mn}$. The recursive optimal transport d_{mn} induces a metric on the integers that allows to study the rate of convergence of (KM). As a result, we settle Baillon and Bruck's conjecture for the rate of convergence of the fixed point residuals: for every non-expansive map in any normed space the following estimate holds with $\kappa = 1/\sqrt{\pi}$

$$\|x_n - Tx_n\| \leq \kappa \frac{\mathrm{diam}(C)}{\sqrt{\sum_{i=1}^{n} \alpha_k(1-\alpha_k)}}.$$

The analysis exploits an unexpected connection with discrete probability and combinatorics, related to the Gambler's ruin for sums of non-homogeneous Bernoulli trials. We will also discuss the extent to which the constant $\kappa = 1/\sqrt{\pi}$ is sharp.

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Submodular financial markets with frictions

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Keywords: Submodularity, financial markets, arbitrage, risk measure, pricing rules, Choquet integral.

This paper characterizes arbitrage-free financial markets with bid/ask spreads that are submodular, i.e., their super-replication cost functions (or super-hedging prices) are submodular, and studies an important class of submodular markets. The submodular assumption on the cost function, or the supermodularity usually assumed on preferences and utility functions, is the formal expression of perfect complementarity, which dates back to Fisher, Pareto, and Edgeworth, according to Samuelson [**?**]. Arbitrage-free markets whose bond is frictionless provide a natural rationale for the famous multi-prior model of Gilboa and Schmeidler [2], extensively used in this paper in its dual form. Actually, the absence of arbitrage opportunities in the market exhibits a family of risk-neutral probability measures, leading to a super-replication cost à la Gilboa-Schmeidler.
Our characterization result is two-fold. First, a market is submodular if and only if its super-replication cost is a Choquet integral and if and only if its set of risk-neutral probabilities is representable as the core of a submodular non-additive probability that is uniquely defined, called risk-neutral capacity. Second, a market is representable by its risk neutral capacity if and only if it is equivalent to a market, only composed of bid/ask event securities, i.e., whose payoffs are characteristic functions of events. Our second contribution is to study an important class of financial markets, only composed of bid/ask event securities. Our main result shows that such a market is submodular if the bond is frictionless and the events (other than the sure event) defining the securities are pairwise disjoint, hence in particular, if they are are bid/ask Arrow securities. The super-replication cost can then be calculated as a Choquet integral with respect to the risk-neutral capacity, which is moreover given by an explicit and tractable formula.

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Integration and approximate subdifferentials calculus for nonconvex functions

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Keywords: Integration, lower semicontinuous convex envelope, epsilon-subdifferential.

We present three integration theorems for the epsilon-subdifferential of nonconvex functions in locally convex spaces. We prove that an inclusion relationship between the epsilon-subdifferentials of any two functions yet yields the equality of the closed convex envelopes up to an additive constant. When this relation only involves small values of epsilon, the integration criterion as well as the conclusion of the integration theorems also take into account the behavior at infinity of the functions. Our analysis relies on subdifferential calculus for some associated functions, namely, lsc, convex, and lsc convex envelopes, as well as the conjugate and the asymptotic functions. Applications concerns extensions of some known properties of convex functions and establishment of optimality conditions in dc optimization.

Nonsmooth critical values and Sard type results

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Keywords: Lipschitz function, Clarke subdifferential, Sard theorem.

We survey recent results of the literature related to Sard type results for nonsmooth critical values of Lipschitz continuous functions. We shall discuss applications to (semi-infinite) optimization. The talk is based on joint works with L. Barbet (Pau), J. Bolte (Toulouse), M. Dambrine (Pau), A.S. Lewis (Cornell, Ithaca) and L. Rifford (Nice).

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ω-limit sets for differential inclusions

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Keywords: Differential inclusion, LaSalle principle, Lyapunov function, ω-limit sets, stabilization, discontinuous feedback, nonsmooth dynamics.

Abstract

This paper is about locating ω-limit sets for solutions of differential inclusions with not necessarily continuous right side. Based on the LaSalle principle we assume that as time $t \to \infty$ the set of solutions approaches a closed subset $\mathcal{S}$ of $\mathbb{R}^n$ and then consider the dynamics restricted on $\mathcal{S}$ to find the location of the ω-limit set by utilizing nonsmooth Lyapunov type functions over a neighborhood of $\mathcal{S}$; then we prove that this location is also valid for the original dynamics. We apply our result for nonsmooth differential equations and compare it with some recent results.

Convex analysis in action: demand functions

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Abstract

Mathematicians are interested in solving convex optimisation problems. Economists observe the solutions of convex optimisation problems, and want to infer the problem from its solution. As a simple example, consider a map $x(p)$ from $\mathbb{R}^n$ into itself. Does there exist a concave function $u(x)$ such that $x(p)$ is the solution of $\max u(y)$ for $py \leq px(p)$? This has been a lifelong research program where I have been engaged with Pierre-André Chiappori, and which has been happily concluded. I will report on some of it.

Tame Variational Analysis

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Keywords: semi-algebraic, subdifferential, stratification, metric regularity, critical point.

Over the past decade, semi-algebraic geometry has had a pronounced impact on optimization and on variational analysis. Semi-algebraic functions – those whose graphs are representable by finitely many polynomial conditions – are common, easy to recognize, and are "pathology-free". The mere existence of polynomial descriptions endows semi-algebraic functions with powerful analytic properties, there to be used. This analytic insight interplays nicely with all the usual notions of variational analysis, such as selection theorems, error bounds, and Sard-type results. In this talk, I will survey the main trends of semi-algebraic variational analysis, assuming no prior familiarity of the audience with the subject.

Finite identification and local linear convergence of proximal splitting algorithms

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Keywords: Forward–Backward, Douglas-Rachford, Partial smoothness, Activity identification, Local linear convergence.

Convex nonsmooth optimization has become ubiquitous in most quantitative disciplines of science. Proximal splitting algorithms are very popular to solve structured convex optimization problems. Within these algorithms, the Forward-Backward and its variants (e.g. inertial FB, FISTA, Tseng's FBF), Douglas-Rachford and ADMM are widely used. The goal of this work is to investigate the local convergence behavior of these schemes when the involved functions are partly smooth relative to the associated active manifolds. In particular, we show that (i) all the aforementioned splitting algorithms correctly identify the active manifolds in a finite number of iterations (finite activity identification), and (ii) then enter a local linear convergence regime, which we characterize precisely in terms of the structure of the involved active manifolds. For problems involving quadratic and polyhedral functions, we show how to get finite termination of Forward-Backward-type splitting. These results may have numerous applications including in signal/image processing, sparse recovery and machine learning. Indeed, the obtained results explain the typical behaviour that has been observed numerically for many problems in these fields such as the Lasso, the group Lasso, the fused Lasso and the nuclear norm regularization to name only a few.

Geometric and topological characetrizations of strong duality in nonconvex optimization with a single equality and geometric constraints

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Keywords: Nonconvex optimization, hidden convexity, strong duality, quadratic programming.

Some topological and geometric characterizations of strong duality for a non convex optimization problem under a single equality and geometric constraints are established. In particular, a hidden convexity of the conic hull of joint-range of the pair of functions associated to the original problem, is obtained. Applications to derive KKT conditions without standard constraints qualification are also discussed. Several examples showing our results provide much more information than those appearing elsewhere, are given. Furthermore, first and second order optimality condition, for a non convex quadratic optimization problem under two quadratic equality constraints, are also presented. Finally, the standard quadratic problem involving a non necessarily polyhedral cone is analyzed in detail.

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Pointwise Second Order Optimality Conditions in Optimal Control

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Keywords: Optimal control; Second-order necessary optimality conditions.

This talk is devoted to the pointwise second-order necessary optimality conditions for the Mayer problem arising in optimal control theory. The control system under consideration involves arbitrary closed, time dependent control sets U(t) and arbitrary closed sets of initial conditions. Optimal controls are supposed to be merely measurable. We first show that with every optimal trajectory it is possible to associate a solution p(.) of the adjoint system (as in the Pontryagin maximum principle) and a matrix solution W(.) of an adjoint matrix differential equation that satisfy a second-order transversality condition and a second-order maximality condition. These conditions seem to be a natural second-order extension of the maximum principle of optimal control.
We apply these results to derive pointwise Jacobson like necessary optimality conditions for general control systems and optimal controls that may be only "partially singular" and may take values on the boundary of control constraints.

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Squared Slack Variables in Nonlinear Second-Order Cone Programming

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Keywords: Second-order cone programming, optimality conditions, slack variables.

A well-known technique in constrained optimization is the introduction of nonnegative slack variables, which converts inequality constraints into equality constraints. It is widely used in *linear programming* and *nonlinear programming* (NLP). The squared slack variables may also be used in NLP, so that the nonnegativity constraints of the variables can be eliminated, as considered, for example, in [1]. However, this strategy is usually avoided in the optimization community, because it increases considerably the dimension of the problem and may lead to numerical instabilities or singularities.
The situation changes in *nonlinear second-order cone programming* (NSOCP). An NSOCP problem is an optimization problem with second-order (or Lorentz) cone constraints, where the involved functions are nonlinear. The use of squared slack variables is more interesting for NSOCP problems than NLP problems. In fact, the reformulated problem with the additional squared slack variables is no longer an NSOCP problem, but only an NLP problem. This is important in practice, especially if one considers the second-order cone as an object that is not so easy to handle. Moreover, this fact indicates that a general-purpose NLP solver may be used to find a solution, or at least a stationary point, of the original NSOCP problem. With this in mind, we will analyze the relation between the original NSOCP problem and the reformulated NLP problem, in terms of Karush-Kuhn-Tucker (KKT) points along with regularity conditions. We will show that the second-order sufficient conditions are the key to establish the equivalence of KKT points. To the authors' knowledge, such analysis has apparently not been published in the literature even in the NLP framework, except for some particular results shown, for example, in [1, Section 3.3].

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Images, Fixed Point and Vector Extrema

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Based on Image Space Analysis, which is here briefly introduced, a new scalarization method for Vector Optimization Problems (VOP) is described. The main feature is a fixed point approach, which leads to formulate a scalar problem of type "quasi-minimum problem". Unlike some existing scalarization methods, the vector of parameters, which scalarizes the objective function, is fixed at the beginning and do not vary during the performance of the method. The particular linear case is briefly outlined. The minimization of a scalar function over the set of solutions to VOP is briefly outlined as a possible development.

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Conjugate Duality and Linear Dynamics with Constraints or Uncertainty

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Keywords: conjugate duality, linear system, convex Lyapunov function, switching system, convex constraint, asymptotic stability, stabilizability and detectability, dissipativity.

As envisioned by Terry Rockafellar in the seminal work *Conjugate Duality and Optimization*, conjugate duality ideas led to advances in, among other areas, the analysis of fully convex problems in calculus of variations and in optimal control. Basic constructions of convex analysis are prominently featured in this analysis: convex conjugate functions define dual problems; saddle functions appear in the Hamiltonian optimality conditions, as partial conjugates of fully convex costs; optimal value functions for a primal and dual control problems turn out to be convex conjugates of one another; etc. The talk will illustrate how these constructions and ideas apply to some questions in control systems theory, beyond optimal control. For example, convex conjugacy between convex Lyapunov and Lyapunov-like functions can be used to deduce duality between properties like asymptotic stability, dissipativity, stabilizability and detectability for linear dynamical systems with constraints or uncertainty.

On the subdifferential mapping of convex integral functions

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Keywords: Convex integrals, Subdifferentials, Calculus rules.

We give explicit characterizations for the exact and the approximate subdifferential mappings of convex integral functions defined on Suslin locally convex spaces. These characterizations involve the approximate subdifferential mappings of the data convex functions defining the associated normal integrand, and do not require any qualification continuity condition on the involved functions, nor special topological or algebraic structures of the index set. Those formulas given exclusively by means of the data functions use the approximate subdifferential (say the epsilon-subdifferential) but with varying amounts of epsilon depending on the associated functions. However, formulas, which use a fixed amount of epsilon, not depending on the function, require the additional term invoking the normal cone to the domain of the integral function. A part of its proper interest, the discrete case corresponding to the sum of countable infinitely many convex functions is studied in order to illustrate the general results and, from the other hand, to cast a bridge between our results and the classical problem dealing with the subdifferential of the sum of finitely many convex functions. This work is within collaboration with A. Jourani and R. Correa.

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Calmness as a constraint qualification for MPECs

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Calmness is an important property of perturbed constraint mappings when deriving dual stationarity conditions in optimization problems, in particular Mathematical Programs with Equilibrium Constraints (MPECs) of the form

$$\min\{\varphi(x, y) | 0 \in F(x, y) + N_\Gamma(y)\},$$

where F is a smooth mapping, Γ is described by a smooth inequality system and 'N' refers to the normal cone. It is well-known [1] that calmness of the mapping

$$p \mapsto \{(x, y) | p \in F(x, y) + N_\Gamma(y)\}$$

ensures the possibility of deriving so-called Mstationarity conditions. Similarly, one may consider the *enhanced* MPEC

$$\min\{\varphi(x, y) \mid 0 \in H(x, y, \lambda) + N_{\mathbb{R}^m \times \mathbb{R}^s_+}(y, \lambda)\}$$

with $H(x, y, \lambda) := \left(F(x, y) + [\nabla q(y)]^T \lambda, -q(y)\right)$. Now, calmness of the enhanced mapping

$$(p_1, p_2) \mapsto \left\{(x, y, \lambda) \,|, (p_1, p_2) \in H(x, y, \lambda) + \hat{N}_{\mathbb{R}^m \times \mathbb{R}^s_+}(y, \lambda)\right\}$$

is required in order to derive the according M-stationarity conditions. The aim of this talk is to compare the calmness properties of both mappings and to provide tools for checking them in different settings.

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Quadratic growth and subregularity of subdifferentials

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Consider the following two properties
(a) $f(x) \geq f(\overline{x}) + k\|x - \overline{x}\|^2$ for x close to $\overline{x}$;

(b) $f(x) \geq f(\overline{x})$ for x close to $\overline{x}$ and ∂f is subregular at $(\overline{x}, 0)$

In 1991 Kummer showed that (a) and (b) are equivalent properties for convex functions on $I\!R^n$ and very recently Aragón Artacho and Geoffroy proved that this is actually true for convex functions on Banach spaces.
The first two results to be reported are:
• implication (b) $\Rightarrow$ (a) holds for any lsc function on any Banach space and any subdifferential trusted on the space (e.g. Fréchet subdifferential on Asplund spaces, Dini-Hadamard subdifferential on Gâteaux smooth spaces and G-subdifferential on any Banach space);
• implication (a) $\Rightarrow$ (b) holds for any lsc and subdifferentially continuous semi-algebraic function on $I\!R^n$ and the limiting subdifferential.
The next result to be discussed shows that (suprisingly!) it is possible to characterize (a) in purely (first order) subdifferential terms without any explicit a priori lower bound for f in a neighborhood of $\overline{x}$.

New Proofs of Maximal Monotonicity and Integrability of the Subdifferential of Convex Function

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Keywords: Subdifferential, Convex Function, Maximal Monotonicity, Integrability.

Let X be a Banach space and let $f : X \to \mathbb{R} \cup \{+\infty\}$ be a proper convex and lower semicontinuous function. It is now classical, but the notion of subdifferential

$$\partial f(x_0) = \{p \in X^*; f(x) \geq f(x_0) + p(x - x_0),\ \forall x \in X\},$$

$\partial f(x_0) = \emptyset$ if $f(x_0) = \infty$; was introduced and established by Moreau and Rockafellar.
This talk presents recent proofs of two properties of this notion:
maximal monotonicity (Rockafellar) and *integrability* (Moreau-Rockafellar).
Maximal monotonicity means that the graph of ∂f can not be properly included in monotone subset of $X \times X^*$, while integrability means

$$\partial g \subset \partial f \Rightarrow f = g + c,$$

where g is proper convex and lower semicontinuous and c is some real constant.
Both proofs use only tools which had been well-established by 1970 like Brøndsted-Rockafellar Theorem for ε-subdifferential.
It is also noteworthy that the proof of integrability [2] uses infimal regularisation like the original proof of Moreau [1].

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Cost-minimizing regulations for an electricity market

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We consider a game-optimization model representing a wholesale electricity market with general networks, transmission losses and strategic producers. Previous works show how regulation mechanisms such as the case when prices correspond to the Lagrange multipliers of a centralized cost minimization program allow the producers to charge significantly more than marginal price. In this paper we consider an incomplete information setting where the cost structure of a producer is unknown to both its competitor and the regulator. We derive an optimal regulation mechanism, and compare its performance to the "price equal to Lagrange multiplier" mechanism in an incomplete information setting, that we solve numerically.

Positively α-far sets and existence results for generalized perturbed sweeping processes

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We consider the general class of positively α-far sets which contains strictly those of uniformly prox-regular sets and uniformly subsmooths sets. We provide some conditions to assure the uniformly subsmoothness, and thus the positively α-farness, of the inverse images under a differentiable mapping. Then, we take advantage of the properties of this class to study the generalized perturbed sweeping process in Hilbert spaces. The later one includes the classical perturbed sweeping process as well as complementarity dynamical systems.

Calm and Locally Upper Lipschitz Multifunctions: Intersection Mappings and Applications in Optimization

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Keywords: Intersection of multifunctions, Calmness, Locally upper Lipschitz multifunction, Argmin mapping, Optimality conditions.

Calmness and upper Lipschitz continuity of multifunctions play a basic role in the stability analysis of optimization problems as well as for deriving optimality conditions and studying solution methods. In this talk, we first recall some basic results from [1] on calmness or locally upper Lipschitz behavior. This includes the characterization by describing mappings, the (more or less classical) application to optimality conditions and exact penalty schemes, and the superlinear convergence of Newton methods. As a main idea we recall basic theorems on calm and locally upper Lipschitz intersections of multifunctions. This is then applied for characterizing, under suitable assumptions, the calmness of the optimal solution set mapping of a parametric program by means of the calmness of an associated parametric system of inequalities. In a recent paper by Canovas et al. [2], theorems of that type were given for the special class of linear semi-infinite programs under canonical perturbations. We will show that some of these results can be extended to a larger class of problems; this is done by combining a basic intersection theorem and classical results from parametric optimization, for details see [3].

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Reconstruction of algebraic-exponential data from moments

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Keywords: Homogeneous polynomial; non-Gaussian integral; Löwner-John ellipsoid

Let $\mathbf{G} \subset \mathbb{R}^n$ be a bounded open subset of Euclidean space with real algebraic boundary $\mathbf{\Gamma}$. In a first part of the talk we consider the case where $\mathbf{G} = \{\mathbf{x} : g(\mathbf{x}) <= 1\}$ for some quasi-homogeneous polynomial $\in \mathbb{R}[\mathbf{x}]$ and derive several properties of $\mathbf{G}$ as well as the non-Gaussian integral $\int \exp(-g)dx$. In particular we show that the volume of $\mathbf{G}$ is a convex function of the coefficients of g.
Next, we consider a more general case and under the assumption that the degree "d" of Γ is given, and the power moments of the Lebesgue measure on $\mathbf{G}$ are known up to order $3d$, we describe an algorithmic procedure for obtaining a polynomial vanishing on Γ. The particular case of semi-algebraic sets defined by a single polynomial inequality raises an intriguing question related to the finite determinateness of the full moment sequence. The more general case of a measure with density equal to the exponential of a polynomial is treated in parallel. Our approach relies on Stokes theorem and simple Hankel-type matrix identities.

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Generic sensitivity analysis for semi-algebraic optimization

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University of Washington

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Technion

Concrete optimization is often semi-algebraic, definable using only polynomial inequalities. The first-order optimality conditions involve a set-valued operator on n-dimensional space whose graph is everywhere n-dimensional (or "thin"). Semi-algebraic monotone operators also have thin graphs, by Minty's theorem. A Sard-type theorem holds for semi-algebraic operators with thin graphs, ensuring good sensitivity behavior for generic data. In particular, optimizers of semi-algebraic problems typically lie on an "active manifold" (identified by popular algorithms), and satisfy strict complementarity and the second-order sufficient conditions.

Outer limit of subdifferentials and calmness moduli in linear and nonlinear programming

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Keywords: Calmness, local error bounds, linear programming, argmin mapping.

With a common background and motivation, the main contributions of this talk are developed in two different directions. Firstly, we are concerned with functions which are the maximum of a finite amount of continuously differentiable functions of n real variables, paying attention to the case of polyhedral functions. For these max-functions, we obtain some results about outer limits of subdifferentials, which are applied to derive an upper bound for the calmness modulus of nonlinear systems. When confined to the convex case, in addition, a lower bound on this modulus is also obtained. Secondly, by means of a KKT index set approach, we are also able to provide a point-based formula for the calmness modulus of the argmin mapping of linear programming problems without any uniqueness assumption on the optimal set. This formula still provides a lower bound in linear semi-infinite programming. Illustrative examples are given.

On the convexity of piecewise-defined functions

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Keywords: Convexity, piecewise-defined function.

Functions that are piecewise defined are a common sight in mathematics while convexity is a property especially desired in optimization. Suppose now a piecewise-defined function is convex on each of its defining components — when can we conclude that the entire function is convex? For example, consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x, y) := \begin{cases} \frac{x^2+y^2+2\max\{0,xy\}}{|x|+|y|}, & \text{if } (x, y) \neq (0, 0); \\ 0, & \text{otherwise.} \end{cases}$$

Clearly, f is a piecewise-defined function with continuous components: $f_1(x, y) := x + y$ on $A_1 := \mathbb{R}_+ \times \mathbb{R}_+$, $f_2(x, y) := \frac{x^2+y^2}{-x+y}$ on $A_2 := \mathbb{R}_- \times \mathbb{R}_+$, $f_3(x, y) := x+y$ on $A_3 := \mathbb{R}_- \times \mathbb{R}_-$, and $f_4(x, y) := \frac{x^2+y^2}{x-y}$ on $A_4 := \mathbb{R}_+ \times \mathbb{R}_-$. One may check that each f_i is a convex function. However, *whether or not f itself is convex* is not immediately clear. (As it turns out, f is convex.)
Our main result provides sufficient conditions for a piecewise-defined function f to be convex.
We also provide a sufficient condition for checking the convexity of a *piecewise linear-quadratic function*, which play an important role in computer-aided convex analysis [1], [3, Section 10.E], [4], and partially answer an open question from [2, Section 23.4.2].

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A Survey of Results on Linear Convergence for Iterative Proximal Algorithms in Nonconvex Settings

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Keywords: alternating projections, constraint qualification, Douglas-Rachford, linear regularity, prox operator, nonconvex feasibility, subregularity, variational analysis.

For iterative methods in nonconvex optimization, a central question is when to stop. And when the decision has been made to stop, what is the relation, if any, between the point that the algorithm delivers and the desired solutions to the optimization problem? At the heart of answers to these questions is the theory of regularity, not only of the underlying functions and operators, but of the set of solutions, and, more generally, critical points. We survey progress over the last several years on sufficient conditions for local linear convergence of fundamental algorithms applied to nonconvex problems, and discuss challenges and prospects for further progress. The theory is local by nature and contains the convex case as an example where the local neighborhood extends to the whole space. The popular affine feasibility problem illustrates that the convex case is not the only instance where global guarantees of convergence of first-order algorithms to globally optimal solutions are possible, and that regularity of the objective function, in conjunction with the constraint structure, is key to global results.

Full Stability of Parametric Variational Systems

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Tran Nghia
Oakland University

In this talk we introduce and characterize new notions of Lipschitzian and Hölderian full stability of solutions to general parametric variational systems described via partial subdifferential and normal cone mappings acting in Hilbert spaces. These notions, postulated certain quantitative properties of single-valued localizations of solution maps, are closely related to local strong maximal monotonicity of associated set-valued mappings. Based on advanced tools of variational analysis and generalized differentiation, we derive verifiable characterizations of the local strong maximal monotonicity and full stability notions under consideration via some positive-definiteness conditions involving second-order constructions of variational analysis. The general results obtained are specified for important classes of variational inequalities and variational conditions in both finite and infinite dimensions.

Non-smooth bundle trust-region method

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Keywords: Descent method, convergence, stopping, subdifferential, trust-region, bundle.

We consider optimization problems of the form

$$\min_{x\in\mathbb{R}^n} f(x), \tag{1}$$

where $f : \mathbb{R}^n \to \mathbb{R}$ is locally Lipschitz but neither smooth nor convex. We show how the trust-region method, which is well-studied in smooth optimization, can be extended to non-smooth non-convex problems (1) when combined with bundling based on cutting planes. We prove global convergence of our novel bundle trust-region method and derive a rigorous and algorithmically useful stopping test.

We then ask the following natural question: How must a subdifferential operator ∂f for locally Lipschitz functions f be designed so that it becomes possible to prove convergence to critical points for the sequences x^j of iterates of suitable non-smooth non-convex descent methods like a non-convex bundle method or the bundle trust-region method? And if such an operator ∂f exists, does it also allow a rigorous stopping test for the descent method in question?

Investigation of Crouzeix's Conjecture via Nonsmooth Optimization

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Keywords: Matrix analysis, field of values, numerical range.

Michel Crouzeix's 2004 conjecture [1] concerns the relationship between $\|p\|_{W(A)}$, the norm of a polynomial p on $W(A)$, the field of values (numerical range) of a matrix A, and $\|p(A)\|_2$, the operator norm of the matrix $p(A)$. We use nonsmooth optimization to investigate the conjecture numerically, using the BFGS (Broyden-Fletcher-Goldfarb-Shanno) method [2] to search for local minimizers of the "Crouzeix ratio" $\|p\|_{W(A)}/\|p(A)\|_2$, computing the boundary of $W(A)$ with Chebfun [3]. The conjecture states that the globally minimal value of the Crouzeix ratio is 1/2. The numerical results lead to some modest theorems and further conjectures about globally and locally minimal values of the Crouzeix ratio when varying only A (with p fixed) or varying only p (with A fixed), or varying over p and A together. All the computations strongly support the truth of Crouzeix's conjecture.

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Stochastic Non-Cooperative Games

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Keywords: Nash equilibrium, stochastic program, risk aversion, nonsmoothness, smoothing, approximation

Short version. This talk presents a Nash equilibrium problem (NEP) under uncertainty formulated as a non-cooperative multi-agent game where each agent solves a stochastic program (SP) under uncertainty parameterized by the rivals' strategies. Challenges of this problem are highlighted and resolutions are proposed.

Extended version. This paper formally introduces and studies a Nash equilibrium problem (NEP) under uncertainty. Mathematically, this problem is formulated as a non-cooperative multi-agent game where each agent solves a stochastic program (SP) under uncertainty parameterized by the rivals' strategies. Several types of uncertainty in the players' optimization problems are highlighted: the first leads to a standard two-stage SP with recourse that models a risk-neutral player who undertakes recourse actions in a second stage where the uncertainty is realized; the second type generalizes the first where mean-deviation or coherent risk measures are employed to describe the players' risk aversion of the recourse decisions; the third type of uncertainty is modeled by the concept of uncertainty sets, leading to a game with (conservative) players solving robust minimax optimization problems; a fourth type of the stochastic NEP is resolved by a tuple of deterministic strategies, one for each player, that constitutes an equilibrium of the game for almost all realizations of the uncertainty. It is noted that each type of such stochastic Nash equilibrium problems (SNEPs) results in the players' optimization problems being non-smooth. Thus, a multi-valued variational inequality is needed to characterize an equilibrium when the players' optimization problems are convex SPs. After these different types of SNEPs are defined, we focus on a broad class of games defined by abstract mean-deviation objectives satisfying certain properties that are inspired by the mean-quantile deviations. Several computational challenges are noted, including: the non-smoothness of the mean-deviation measures composite with the expectation of the recourse functions; the numerical evaluation of the expectation operator, and the expected absence of monotonicity in the resulting (multi-valued) variational formulation. To handle the non-smoothness of the mean-deviation measures, we propose smoothing schemes that lead to differentiable approximations; to handle the recourse functions, we apply a pull-out scheme whereby additional (fictitious) players are employed who optimize the recourse objectives subject to uncertainty-realized constraints that may couple the players' strategies; to deal with non-monotonicity, we impose diagonal dominance on the players' smoothed objective functions that facilitate the application and convergence of an iterative best-response scheme; to handle the expectation operator, we rely on known methods in stochastic programming that include sampling and approximation as well as a progressive hedging scheme. Overall, this paper lays the foundation for future research into the class of SNPEs that provides a constructive paradigm for solving competitive decision making problems under uncertainty; this paradigm is very much at an infancy stage of research and requires extensive treatment in order to meet its broad applications in many engineering and economics domains.

Duality and optimality in stochastic optimization and mathematical finance

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Ari-Pekka Perkkiö
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Keywords: Convex stochastic optimization, duality, mathematical finance

This article studies convex duality in stochastic optimization over finite discrete-time. The first part of the paper gives general conditions that yield explicit expressions for the dual objective in many applications in operations research and mathematical finance. The second part derives optimality conditions by combining general saddle-point conditions from convex duality with the dual representations obtained in the first part of the paper. Several applications to stochastic optimization and mathematical finance are given.

Regularity and regularization in variational analysis

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The word “regularity” has many different meanings in mathematics. In nonsmooth analysis the concepts used by F.H. Clarke [C], R.T. Rockafellar [R2, R3], and J.-P. Penot [P1] (under the terms of “regularity”, “protodifferentiability”, and “softness” respectively) can be completed by a number of different notions. The purposes of such concepts are twofold: first, for restricted classes of functions, they reduce the number of available subdifferentials; second, they enable to get new properties, in particular equalities instead of inclusions.

Given two subdifferentials ∂_A and ∂_B, we suggest to say that a function f is A-B-regular at some point x of its domain if $\partial_A f(x) = \partial_B f(x)$. Of course, if f is convex or approximately convex, or of class C^1, f is A-B-regular for all usual subdifferentials ∂_A and ∂_B. Since the calculus rules for ∂_A and ∂_B may be different, in some cases of interest A-B-regularity can be transferred to new functions build from f.

This talk is focused on the application of such ideas to integral functionals, evoking the impressive recent work of E. Giner showing that F-I-regularity (F for firm or Fréchet and I for incident or adjacent or intermediate) can be transferred from an integrand satisfying a certain growth condition to the corresponding integral functional on some Lebesgue space. Under such conditions, the integral functional associated with a (nonconvex) Legendre function is a Legendre function and its Legendre transform is the integral functional associated with the Legendre transform of the integrand. Such results can be seen as extensions of the pioneering studies of convex integral functionals made by R.T. Rockafellar a long time ago in [R1].

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Model Risk: new techniques

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The model risk in finance is the risk of potential loss resulting from making decisions using mathematical models for valuation, risk measuring, or for capital economic computation. More precisely, according to Supervisory Guidance on Model Risk Management (FED [1]) model risk occurs primarily for two reasons: (1) a model may have fundamental errors and produce inaccurate outputs when viewed against its design objective and intended business uses; (2) a model may be used incorrectly or inappropriately or there may be a misunderstanding about its limitations and assumptions. This talk is devoted to present the new techniques to assess the model risk. To do this, we will introduce the concept of superior model versus inferior model. Many examples are given in order to illustrate our new approach to assessing model risk.

References

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Projections and the Reduction Lemma

Stephen M. Robinson
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Keywords: Variational inequality, polyhedrality, critical cone, normal manifold.

The Reduction Lemma, aptly named by Asen Dontchev and Terry Rockafellar in 1996, shows that near a solution point of a variational inequality posed over a polyhedral convex set, the variational inequality can be taken to be posed over a polyhedral convex cone (the critical cone associated with the solution). It is a key to understanding why the critical cone is so important for the local analysis of solutions. In this lecture we will show how to combine some properties of projections with the geometry of the normal manifold of a polyhedral convex set to produce a simple and geometrically intuitive proof of the Reduction Lemma.

Measures of Residual Risk with Connections to Regression, Risk Tracking, Surrogate Models, and Ambiguity

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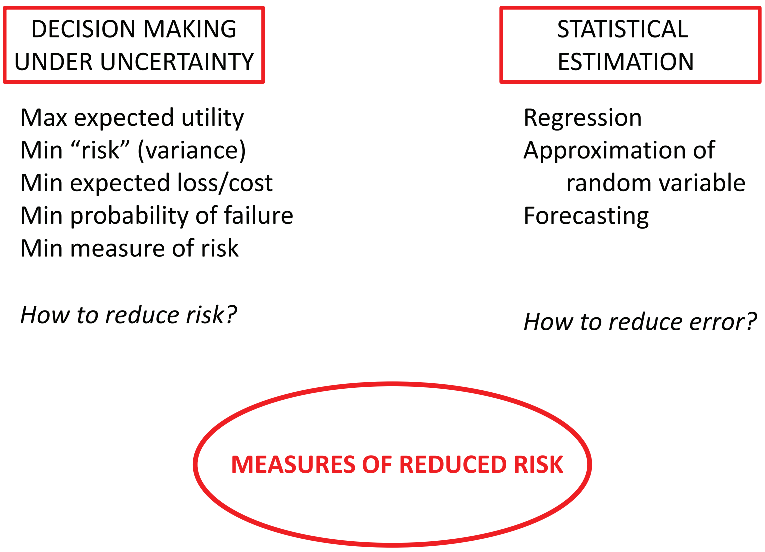
Keywords: risk measures, risk quadrangle, decision making under uncertainty, regression, stochastic ambiguity.

Measures of residual risk are developed as extension of measures of risk. They view a random variable of interest in concert with an auxiliary random vector that helps to manage, predict, and mitigate the risk in the original variable. Residual risk can be exemplified as a quantification of the improved situation faced by a hedging investor compared to that of a single-asset investor, but the notion reaches further with deep connections emerging with forecasting and generalized regression. We establish the fundamental properties in this framework and show that measures of residual risk along with generalized regression can play central roles in the development of risk-tuned approximations of random variables, in tracking of statistics, and in estimation of the risk of conditional random variables. The presentation ends with dual expressions for measures of residual risk, which lead to further insights and a new class of distributionally robust optimization models.



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An approximation scheme for a class of risk-averse stochastic equilibrium problems

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Juan Pablo Luna
UFRJ, Rio de Janeiro, Brazil

Mikhail Solodov
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We consider two models for stochastic equilibrium: one based on the variational equilibrium of a generalized Nash game, and the other on the mixed complementarity formulation. Each agent in the market solves a one-stage risk-averse optimization problem with both here-and-now (investment) variables and (production) wait-and-see variables. A shared constraint couples almost surely the wait-and-see decisions of all the agents. An important characteristic of our approach is that the agents hedge risk in the objective functions (on costs or profits) of their optimization problems, which has a clear economic interpretation. This feature is obviously desirable, but in the risk-averse case it leads to variational inequalities with set-valued operators – a class of problems for which no established software is currently available. To overcome this difficulty, we define a sequence of approximating differentiable variational inequalities based on smoothing the nonsmooth risk measure in the agents' problems, such as average or conditional value-at-risk. The smoothed variational inequalities can be tackled by the PATH solver, for example. The approximation scheme is shown to converge, including the case when smoothed problems are solved approximately. To assess the interest of our approach, numerical results are presented. The first set of experiments is on randomly generated equilibrium problems, for which we show the advantages of our approach when compared to the standard smooth reformulation of minimization involving the max-functions (such as the average value-at-risk). The second set of experiments deals with part of the real-life European gas network, for which Dantzig-Wolfe decomposition is combined with the smoothing approach.

Newton-Type Methods: A Broader View

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Keywords: Newtonian methods, Generalized equations, Constrained optimization.

We discuss abstract Newtonian frameworks for generalized equations, and how a number of important algorithms for constrained optimization can be related to them by introducing structured perturbations to the basic Newton iteration. This gives useful tools for local convergence and rate-of-convergence analysis of various algorithms from unified perspectives, often yielding sharper results than provided by other approaches. Specific constrained optimization algorithms, that can be conveniently analyzed within perturbed Newtonian frameworks, include the sequential quadratic programming method and its various modifications (truncated, augmented Lagrangian, composite step, stabilized, and equipped with second-order corrections), the linearly constrained Lagrangian methods, inexact restoration, sequential quadratically constrained quadratic programming, and certain interior feasible directions methods. We recall most of those algorithms as examples to illustrate the underlying viewpoint. We also discuss how the main ideas of this approach go beyond clearly Newton-related methods and are applicable, for example, to the augmented Lagrangian algorithm (also known as the method of multipliers), which is in principle not of Newton type since its iterations do not approximate any part of the problem data.

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Finite composite games: equilibria and dynamics

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Keywords: composite games, equilibria, dynamics

We introduce finite games with the following types of participants:
(I) nonatomic players,
(II) atomic splittable players,
(III) atomic non splittable players.
We recall and compare the basic properties, expressed through variational inequalities, concerning equilibria, potential games and dissipative games, as well as associate evolutionary dynamics.
Then we consider composite games, a typical example being congestion games, and extend the previous properties of equilibria and dynamics.
Finally we describe an instance of composite potential game.

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A distribution-ambiguous scheme for multi-stage stochastic optimization

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Keywords: risk measure, robust optimization, stochastic optimization, stochastic variational inequality.

Abstract : Solving stochastic optimisation problems often requires full information on distribution of the random variables involved. When such information is not available, an alternative scheme is to use the distribution-ambiguous optimization model that only requires partial information on the distribution. It turns out that this approach helps to overcome curse of dimensionality since under reasonable assumptions the deterministic equivalence is a conic optimization problem whose size is not proportional to the number of scenarios. Another advantage of this model is that the theoretical framework of distribution-ambiguous optimization naturally fits into risk measure theory, therefore becomes a special class of risk measure optimization and is closely related to stochastic variational inequality, both of which have been recently studied by Rockafellar and his collaborators.

Three Rockafellar Theorems

Lionel Thibault
University of Montpellier

The talk will present various ways opened by three Rockafellar theorems as well as various applications.

Martingale Optimal Transport and Robust Hedging

Nizar Touzi
Ecole Polytechnique, Paris

Keywords: Optimal transport, quasi-sure formulation, Skorohod embedding, robust hedging.

The martingale optimal transport problem is the dual the robust hedging of exotic derivatives under marginals constraints. The relation of these two problems is the analogue of the Kantorovitch duality in the standard Monge-Kantorovitch optimal transport problem. We provide an overview of the existing results in this recent topic, and we emphasize the connection with the Skorohod embedding problem.

Buffered Probability of Exceedance: Mathematical Properties and Applications

Stan Uryasev
University of Florida

Matt Norton and Alexander Mafusalov
University of Florida

Keywords: CVaR, Conditional Value at Risk, Probability of Excedance, POE, Buffered Probability of Exceedance, bPOE, AUC, Classification

The probability of exceedance (POE) is frequently used to measure uncertainties in outcomes. For instance, POE is used to estimate probability that assets of a company fall below liabilities. POE measures only the frequency of outcomes and ignores magnitude of outcomes. POE counts outcomes exceeds the threshold, and it "does not worry" about the amount by which each outcome exceeds the threshold. POE is lumping together all threshold exceedance events, potentially "hiding" quite large and very troublesome outcomes. Moreover, POE has poor mathematical properties when used to characterize discrete distributions of random values, e.g., when distributions are defined by previously observed historical data. POE for discrete distributions is a discontinuous function of control variables, making it difficult to analyze and optimize.

This paper investigates a new probabilistic characteristic called *buffered probability of exceedance (bPOE)*. With bPOE, it is possible to count outcomes similar to a threshold value, rather than only outcomes exceeding the threshold. To be more precise, bPOE counts tail outcomes averaging to some specific threshold value. For instance, 4% of land-falling hurricanes in US have cumulative damage exceeding \$50 billion (i.e., POE = 0.04 for threshold=\$50 billion). It is estimated, that the average damage from the worst 10% of hurricanes is \$50 billion. In terms of bPOE, we say bPOE=0.1 for threshold=\$50 billion. bPOE shows that largest damages having magnitude around \$50 billion have frequency 10%. bPOE can be considered as an important supplement to POE. We think that bPOE should be routinely calculated together with POE. This example shows that bPOE exceeds POE, which is why it is called Buffered Probability of Exceedance. The positive difference, bPOE-POE, can be interpreted as some "buffer." The bPOE concept was recently developed as an extension of Buffered Probability of Failure (introduced by Rockafellar and Royset). bPOE has been derived from Conditional Value-at-Risk (CVaR) characteristic of uncertainty. Actually, bPOE is an inverse function of CVaR and it inherits a majority of the exceptional mathematical properties of CVaR (which is a so called "coherent measure of risk"). Similar to CVaR, minimization of bPOE can be reduced to convex and Linear Programming.

We will discuss two applications of bPOE concept. The first application considers the Cash Matching of a Bond Portfolio. We minimize bPOE that assets exceed liabilities. The second application uses bPOE in data mining. Currently, the Area Under the Receiver Operating Characteristics Curve (AUC) is standardly used to evaluate classification models. AUC can be presented as the probability that some discrete random value is below zero. We explored so called Buffered AUC (bAUC) as a counterpart of the standard AUC. Download Reports:

- `www.ise.ufl.edu/uryasev/files/2015/02/bAUC_workingPaper.pdf`
- `www.ise.ufl.edu/uryasev/files/2011/08/bPOE_bSL_final.pdf`
- `www.ise.ufl.edu/uryasev/files/2011/08/buffered_probability_of_exceedance.pdf`

Regularity Properties of the Hamiltonian in Optimal Control

Richard Vinter
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Keywords: Differential Inclusions, Necessary Conditions, Bounded Variation, Regularity, State Constraints.

It is well known that the Hamiltonian, evaluated along the optimal trajectory and costate arc, is constant. In this talk we observe that this is just one manifestation of the fact that the Hamiltonian inherits certain regularity properties of the data (with respect to time). Special attention is given to the regularity properties of the Hamiltonian, when state trajectories are required to satisfy a pathwise state constraint and when the data is merely of bounded variation. In this case, new estimates are given on the cumulative variation of the Hamiltonian, in terms of the cumulative variation of the data. The estimates lead to improved conditions for non-degeneracy of the state constrained Maximum Principle, and also for the Lipschitz continuity of minimizers.

Resolvent Averages of Monotone Operators in Hilbert Spaces

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Keywords: Dominant property, graphical convergence, monotone operator, paramonotone, rectangular, resolvant average, recessive property.

Monotone operators play important roles in optimization and convex analysis. We define a new average of monotone operators by using resolvants. The new average enjoys self-duality and inherits many nice features of given monotone operators. When the monotone operators are positive definite matrices, the new average lies between the harmonic average and arithmetic average. Appropriate limits of resolvant average lead to both harmonic average and arithmetic average.

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A tale about functions of two variables.

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This lecture is a tale about the evolution of the analysis of bifunctions (bivariate functions) which at the outset was mostly concerned with duality in the convex-concave case and approximation issues. It was initiated by R.T. Rockafellar in the last chapters of "Convex Analysis" (1970) and, subsequently, by H. Attouch and myself "A convergence theory for saddle functions" (1983). One critical issue that hampered seriously some potential applications was a lack of uniqueness both when the saddle function is obtained via partial conjugacy of a convex bifunction or as the epi/hypo-limit of a collection of saddle functions.
J.-P. Aubin, working at the time on the relationship between existence results for a variety of variational problems, suggested that we should expand our convergence results to the family of bifunctions where one would only be interested in the convergence of their maxinf-points rather than, the usually nonexistent, saddle points. Again, in collaboration with H. Attouch, we introduced the notion of lopsided convergence (1983) providing as example how it could be used to design approximates of fixed point problems.
Interest was reawakened, at the beginning of this century, when with A. Jofré, we got interested in stability issues related to Walras equilibria (2001). It turns out that such equilibrium points are maxinf-points of a particular bifunction. This led us to identify a slew of variational problems (variational inequalities, co-operative and non-cooperative games, Nash equilibrium points, solution of generalized equations, MPEC problems and so on) whose solutions can be identified with the maxinf-points of appropriately defined bifunctions. However, the initial Rockafellar paradigm, followed in the epi/hypo-convergence theory, no longer fitted very well this "new interesting" family of bifunctions. This eventually brought about adjusted definitions of lopsided- and epi/hypo-convergence as well as a revised bifunctions-framework. In carrying out this work we have profited significantly from the contributions of some of our students and associates: D. Azé, A. Bagh, S. Lucero, H. Riahi, M. Casey, M. Soueycatt, M. Ait Mansour, J. Deride, P.Q. Khanh and J. Royset.

Essential velocities in stratified control systems

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Keywords: differential inclusions, stratified systems, essential velocities.

We study the minimal time problem for a control system whose state space exhibits a Whitney-like stratification. The overall system is thus discontinuous but is well behaved on each strata. The issue we address is a characterization of those velocities that are the actual derivatives of a state trajectory that leaves one strata and enters another. A key assumption is that the closure of each strata is proximally smooth and relatively wedged.

Necessary optimality conditions for optimal control problems with equilibrium constraints

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We introduce and study the optimal control problem with equilibrium constraints (OCPEC). The OCPEC is an optimal control problem with mixed state and control constraints formulated as time dependent complementarity constraints and it can be seen as a dynamic mathematical program with equilibrium constraints (MPEC). It provides a powerful modeling paradigm for many practical problems such as bilevel optimal control problems and dynamic principal-agent problems. In this paper, we propose several stationary conditions for the OCPEC such as the Clarke (C-) stationarity, Mordukhovich (M-) stationarity, and strong (S-) stationarity in line with the C-, M-, and S-stationarities for the MPEC in the literature. Moreover, we give some sufficient conditions to ensure that the local minimizers of the OCPEC are C-, M-, and S-stationary, respectively.

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